Report on the Experiments with Random Number Generation

Uniformity Test

The uniformity test which is designed to check whether the Ui’s appear to be uniformly  
distributed between 0 and 1, and it is a special case of the chi-square test with all parameters  
known.

Here we see that only one value got rejected where N = 500 and k = 20. Though in higher values  
of N we didn’t get rejection so it can be said that this value got rejected because of some anomaly in the generator.

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| --- | --- | --- | --- | --- |
| K | Value of N | *X2* | ***X2k-1,1-***α | Verdict |
| 10 | 20 | 14.683 | 6.5 | ACCEPTED |
| 500 | 14.683 | 58.02 | REJECTED |
| 4000 | 14.683 | 408.3725 | REJECTED |
| 10,000 | 14.683 | 1005.685 | REJECTED |
| 20 | 20 | 27.203 | 12.0 | ACCEPTED |
| 500 | 27.203 | 37.44 | REJECTED |
| 4000 | 27.203 | 224.875 | REJECTED |
| 10,000 | 27.203 | 514.274 | REJECTED |

**Runs Test**

The *runs* (or *runs-up*) *test*, is a more direct test of the independence assumption. We examine  
the *Ui* sequence (or, equivalently, the *Zi* sequence) for unbroken subsequences of maximal length within which the *Ui*’s increase monotonically; such a subsequence is called a *run up*. Runs tests look solely for independence and not specifically for uniformity.

Here we can see that when the number generated is too high then the null hypothesis got rejected. Because when the value of N is high then they do not remain that much independent.  
And we get high lengths *run up.* Consequently, they get dependent in a way.

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| Value of N | *R* | ***X26,1-***α | Verdict |
| 20 | -567.387 | 10.645 | ACCEPTED |
| 500 | 1153.62 | 10.645 | REJECTED |
| 4000 | 960.172 | 10.644 | REJECTED |
| 10,000 | -1590.954 | 10.644 | ACCEPTED |

**Serial Test**

The *serial test* is really just a generalization of the chi-square test to higher dimensions. If the  
*Ui*’s were really IID U(0, 1) random variates, the nonoverlapping *d*-tuples **U**1 (*U*1, *U*2, . . . , *Ud*), **U**2  
(*Ud*+1, *Ud*+2, . . . , *U*2*d*), . . .should be IID random *vectors* distributed uniformly on the *d* dimensional unit hypercube, [0, 1]*d*.

Here we see that we got one rejections where the values of N, d, k were highest. So, it can be  
said that due to higher dimensions or larger intervals our hypothesis got rejected.

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| Value of N | Value of d,k | *X2* | ***X2k^d-1,1-***α | Verdict |
| 20 | d=2,k=4 | 9.2 | 22.307 | ACCEPTED |
| d=2,k=8 | 54 | 77.745 | ACCEPTED |
| d=3,k=4 | 58 | 77.745 | ACCEPTED |
| d=3,k=8 | 506 | 552 | ACCEPTED |
| 500 | d=2,k=4 | 9.712 | 22.307 | ACCEPTED |
| d=2,k=8 | 70.0 | 77.745 | ACCEPTED |
| d=3,k=4 | 60.699 | 77.745 | ACCEPTED |
| d=3,k=8 | 463.205 | 552.374 | ACCEPTED |
| 4000 | d=2,k=4 | 13.904 | 22.307 | ACCEPTED |
| d=2,k=8 | 67.776 | 77.745 | ACCEPTED |
| d=3,k=4 | 49.41 | 77.745 | ACCEPTED |
| d=3,k=8 | 520.263 | 552.374 | ACCEPTED |
| 10000 | d=2,k=4 | 6.982 | 22.307 | ACCEPTED |
| d=2,k=8 | 62.093 | 77.745 | ACCEPTED |
| d=3,k=4 | 42.947 | 77.745 | ACCEPTED |
| d=3,k=8 | 560.074 | 552.374 | REJECTED |

**Correlation Test**

The correlation test is a direct way to assess whether the generated *Ui*’s exhibit discernible  
correlation: Simply compute an estimate of the correlation at lags *j* 5 1, 2, . . ., *l* for some value  
of *l*. The test should probably be carried out for several values of *j*, since it could be, for instance,  
that there is no appreciable correlation at lags 1 or 2.

but there is dependence between the *Ui*’s at lag 3, due to some anomaly of the generator.

Here we can see that for different values of *j* we didn’t get any rejection. So, there’s no correlation  
between the numbers generated at different lags. As a result, it can be said that the numbers  
that have been generated resemble values of true IID U(0, 1) random variates.

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| Value of j | Value of N | |Aj| | Z1-a/2 | Verdict |
| 1 | 20 | 0.7956 | 1.644853 | ACCEPTED |
| 500 | 0.3762 | 1.6448 | ACCEPTED |
| 4000 | 0.6699 | 0.0627 | ACCEPTED |
| 10,000 | 0.5500 | 1.64485 | ACCEPTED |
| 2 | 20 | 0.2129 | 1.64485 | ACCEPTED |
| 500 | 0.6515 | 1.64485 | ACCEPTED |
| 4000 | 0.97893 | 1.64485 | ACCEPTED |
| 10,000 | 0.83346 | 1.64485 | ACCEPTED |
| 3 | 20 | 0.174586 | 1.64485 | ACCEPTED |
| 500 | 0.41930 | 1.64485 | ACCEPTED |
| 4000 | 0.265994 | 1.64485 | ACCEPTED |
| 10,000 | 1.09899 | 1.64485 | ACCEPTED |